Determining The Securities Market Line Equation in the Philippine Market Using Polynomial Regression

Aaron Rian Choi[1], Brian Godwin Lim[1], Terence Brian Tsai[1]
[1]*Mathematics Department, School of Science and Engineering, Ateneo de Manila University*

Abstract—This study aimed to determine the securities market line (SML) equation in the Philippine market using three polynomial regression. First, in the simple linear model, the SML equation obtained was $\mu_{r_i} = -16.58\% + 25.98\% \times \beta_i$. Hypothesis testing was then done to the market risk premium obtained and the result show that the beta of a security has an effect on its return. This model has a coefficient of determination of 0.127 which indicates that the linear regression model is not useful. Second, a quadratic regression was done and the SML equation obtained was $\mu_{r_i} = 6.50\%\beta_i^2 + 16.39\%\beta_i - 13.71\%$. In the quadratic regression, the market exhibits an increasing marginal return. Finally, a cubic regression was done and the SML equation obtained was $\mu_{r_i} = 103.21\%\beta_i^3 - 223.20\beta_i^2 + 164.67\%\beta_i - 39.70\%$. In the cubic regression, the market exhibits a decreasing marginal return when the asset is less risky and an increasing marginal return when the asset is risky. Theoretically, the analysis could be extended to an even higher degree polynomial but for practical purposes, a cubic regression will suffice.

I. Introduction

This paper aims to apply the Capital Asset Pricing Model in the context of the Philippine Market. Here, we try to determine the equation of the security market line for the Philippine Stock Market, and test whether it can or cannot be used as a model to the actual returns provided.

The Capital Asset Pricing Model (CAPM) is one of the most widely accepted theories in Modern Portfolio Theory. It describes the relationship between the systematic risks and required rate of return of certain assets, mostly stocks. CAPM is theoretically used to acquire the appropriate return on investment, given the risks of the stock, considering also the average return of the whole market. This helps determine whether the addition of a certain stock would or would not be beneficial and helps make a decision on whether to invest in it or not.

While some papers published credit the creation of the Capital Asset Pricing Model to financial economist John N Treynor because of the manuscripts he created namely, "Market Value, Time, and Risk" (1961) and "Toward a Theory of Market Value of Risky Assets" (1962), however, the Treynor Model of CAPM did not get publicized and is a different model from the one people use today. The creation of the Capital Asset Pricing Model, the single period discrete Capital Asset Pricing Model being used today has historically been credited to the works of William F. Sharpe, and John Lintner [1].

Sharpe and Lintner, building on the earlier work of Harry Markowitz on portfolio theory, independently developed their own versions of CAPM in the start of the 1960s. In 1990, Sharpe's work on the development of CAPM together with the work of Harry Markowitz and Merton Miller gave them a shared Nobel Prize for economics "for their pioneering work in the theory of financial economics" [2].

The work of Sharpe and Lintner on the Capital Asset Pricing Model was the beginning of asset pricing theory. Almost 60 years since it was first published in an academic journal, the CAPM still remains as one of the most widely used and taught models in finance. It is used in many different areas such as portfolio creation and estimating the WACC or weighted cost of capital of firms. It remains as the central idea to many of the investment courses in the world and even with its age, remains as one of the most relevant thoughts in Financial Economics [3].

The beauty of the CAPM is that it is able to capture the risk factor of a certain asset, such as a bond or a stock, and translate this amount of risk into the required rate of return that would be justifiable given said amount of risk. It is able to provide investors with a simple, linear relationship between the market risk, the market risk premium and the required rate of return of a single stock. It is very powerful, yet very intuitive and easy to understand in nature which makes it very useful for the people involved in investment theory.

The Sharpe-Lintner CAPM, as mentioned above, builds on the theory of portfolio choice by Harry Markowitz which aims to either minimize variance given expected return or maximize expected return given a set variance. Several things are assumed under the Sharpe-Lintner model, first is that there exists a risk-free asset with a known and constant return, and secondly, all investors can borrow or invest at the same constant return as the risk-free asset. With these assumptions, under the Sharpe-Lintner model, the expected return of an asset is determined by taking the sum of the premium for risk of a certain asset, and the return of the risk-free asset.

Under the Sharpe-Lintner CAPM, the expected return of a single asset is obtained by the following formula:

$$r_a = r_{rf} + \beta_a(r_m - r_{rf}),$$

where r_{rf} is the rate of return for a risk-free security, r_m is the broad market's expected rate of return, and β_a is the beta of the asset a.

Further improvement of the Capital Asset Pricing Model came with the development of the Black CAPM by Fischer Black in 1972. Fischer Black noticed that with empirical tests, that the actual securities market line (SML) supposed to be flatter than that of the Sharpe-Lintner Model. In order to correct this model, Fischer Black created his own CAPM with his own assumptions.

Under the Black CAPM, the assumption is that instead of being able to borrow at the risk-free rate, the investor is allowed to obtain the same results of a Mean-Variance Efficient portfolio by being allowed unlimited short-selling of stock. The Black CAPM is also called the zero beta CAPM since the short selling makes it possible to obtain a beta of 0. The expected return is also adjusted with response to the zero-beta premium. This adjustment lead to a flatter slope for the SML and made the model more robust against future empirical tests [4].

The expected return of a single asset under the Zero Beta CAPM is given as follows:

$$r_e = r_z + \beta_e(r_m - r_z),$$

where r_z is the rate of return for a risk-free security plus the zero-beta premium, r_m is the broad market's expected rate of return, and β_e is the beta of the asset e.

This development of a model that is more robust to empirical testing was essential to the expansion of the reach and influence of the CAPM in both the financial industry and the academe.

Although the Fischer Black model proved to be more robust to empirical testing, in this paper, we aim to analyze the SML created under the Sharpe-Lintner model as one of the key assumptions in the Fischer Black Model, that is that there is no constraint in short selling of assets is not satisfied as short selling of stocks is not permitted in the Philippines. Moreover, although the Fischer Black Model lead to the rise of CAPM as a widely accepted and used theory in Finance, the more commonly used type of CAPM is the Sharpe-Lintner CAPM and thus it would be more relevant to test this model in the context of the Philippine Market.

II. Review of Related Literature

The Capital Asset Pricing Model provides a good benchmark of the market and also distinguishes between diversifiable and non-diversifiable risks. The model assumes that there is one source of systematic risk which is the market risk that affects expected returns. From the Capital asset pricing model, the security market line can be derived and gives a basis on whether a stock will perform well relative to the market. However, the security market lines vary depending on the index that it is compared to. In order to remove this, the security market line must be compared to an agreed upon market index. Furthermore, not all models generated by this method are valid financial models since it varies from market to market. These established strategies have been used previously in generating a model for the New York Stock Exchange, Emerging markets in Central and Southeastern Europe, Chinese markets, and global aggregate stock market prices [5].

New York Stock Exchange

The goal of this study was to generate a general equilibrium model for the pricing of capital assets. In generating the capital asset pricing model, it was assumed that: all investors chose portfolios to maximize returns based only on the mean and variance, tax and transaction costs were ignored and all investors can borrow or lend with a given riskless rate of interest. These assumptions led to a model that encapsulates the relation between risk premiums on assets against systematic risk.

The data used in generating the model was obtained from the University of Chicago Center for Research in Security Prices Monthly Price Relative File. This file contained the monthly prices and dividends for all stocks listed on the New York Stock Exchange during the period 1926-1966. The monthly returns of the market were constructed to be the returns of a portfolio that consists of equal investments in all stocks listed on the New York Stock Exchange. In order to better process this data, each security was placed into ten different portfolios such that each portfolio contained a large spread in its $\beta's$. The 30-day rate on U.S. Treasury Bills for the period 1948-1966 was used as the risk-free rate of the CAPM. From this data, multiple models were generated, each of which were tested for a β factor.

The capital asset pricing model generated from this data showed that high-beta securities had significantly negative intercepts while low-beta securities had significantly positive intercepts. This result is contrary to the predictions of the CAPM [6].

Central and Southeastern Europe

The goal of this study was to examine whether the CAPM is a useful model for capital asset valuation of the emerging securities markets of Central and South-Eastern Europe, to measure whether β is an estimator of the risks of this emerging market, as well as to determine whether the stock market indexes lie on the efficient frontier. In generating the CAPM for this region, assumptions similar to the study done on New York Stock Exchange were held.

The data used for this model was obtained by selecting the securities according to their weights in the stock market indices. Specifically, the ten most liquid stocks were the ones chosen for this model. The model generated was for the period 2006-2010. Furthermore, the benchmark market portfolio used was the official stock market index.

The CAPM generated from this sample was determined to be not useful in pricing the capital assets of the emerging markets in Europe. Furthermore, it was also tested that higher yields do not imply a higher β, which is inconsistent with the predictions of the CAPM model. It was also discovered that the stock market indices do not lie along the efficient frontier. Thus, the indices for these emerging markets are not accurate models of the market [7].

China and Shenzhen

This study aims to test validity of the CAPM for the rising China capital market. The same assumptions as the previous studies were held except for the additional assumption that investors are price acceptors. In order to generate a substantial model, the study took note of sixty-five securities from the Shanghai and Shenzhen stock market. These sixty-five data points were obtained by a random sampling from the complete list of securities in the Shanghai Stock Exchange. The market return rate was obtained by using the Shanghai Stock Index and Shenzhen Component index. The study took the return rate of each stock at daily closing price for the year 2007.

The models obtained were rigorously tested for consistency with the predictions of CAPM. First, it was determined that the β coefficient obtained was significant and as such meant that risk and return were related by the constant β. Secondly, the resulting model showed increasing risks in response to increasing returns which is in agreement with the predictions of the CAPM. Lastly, the significance of the fitted line was tested, and the model was determined to be useful [8].

Singapore

The aim of this study was to determine whether a relationship between average returns and β exists. This is to test the validity of CAPM in the Singapore market, as the CAPM predicts that securities with high β's would imply higher average returns. Furthermore, the CAPM assumptions of the studies above was also used for this study.

The study used data from the Stock Exchange of Singapore to generate the CAPM. A total of sixty securities were chosen from the market. These sixty stocks were chosen from the total of eighty stocks available on the market. These were chosen by continuously listing down each stock that had at least one trade record in a month for the period of 1986-1996 (a total of 132 months). The market return rate was pinned to be the value of the return rate of the All-share index.

The results of the study show that for the Singapore securities market, the CAPM is not a good model. This is because the results from the study did not show a positive relation between risk and reward. Thus, contradicting the predictions of the CAPM [9].

Global

The aim of this study was to analyze the performance of the CAPM on a global scale. This study was done on eighty countries over a period of fifty years. This was done by first obtaining the real returns and country portfolio $\beta's$, which were assumed to be constant over the period of the study. Then using the beta estimates, β, across country portfolios to obtain cross-section regression.

The data used for this study was collected from 82 countries by obtaining their respective national equity data. These countries were chosen in such a way that the country portfolio contained: 23 developed, 36 emerging, and 23 frontier markets. The Morgan Stanley Capital International aggregate indexes were used for the three groups mentioned above. The country stock market indexes were chosen to be the largest security by market share. The Morgan Stanley Capital International All Country World Index was used as the market return of the entire country portfolio. In addition to this, the ten-year government bonds of thirty-four countries were also used. Lastly, the global risk-free rate was pinned to the 3-month U.S. Treasury Bill return. The data points from the sample above was collected monthly over the period 1968-2017.

The study was able to conclude that in the short run, the positive relation between risk and reward may breakdown during times of high volatility. Moreover, the results also suggest that there is a lower systematic risk in frontier markets when compared to both developing and developed markets [10].

Summary

The five studies on the CAPM on the New York Stock Exchange, European securities markets, Shanghai and Shenzhen stock exchange, Singapore stock exchange, and the global securities market were consistent in how the data for the CAPM was gathered. However, the conclusions of the five studies are grouped into two main categories: Predictions of CAPM are consistent, Predictions of CAPM are incorrect. Of the five studies, the CAPM models obtained from the China stock exchange and the global study was found to be consistent. The rest of the CAPM studies on the U.S., Central and South-Eastern Europe, and Singapore resulted in the CAPM having inconsistent results with its predictions.

III. Methodology

In this paper, we studied the behavior of some securities in the Philippine stock market and attempted to obtain a securities market line (SML) equation from these. The securities chosen came from six different sectors namely: holdings, financial, property, industrial, mining, and services. In each of these six sectors, we chose 5 securities each with varying risks as denoted by their beta β. After obtaining the data, we performed polynomial regression to analyze the overall behavior of the market, specifically, how the beta of a security affects its return. First, we performed a simple linear regression. This will give us a linear relationship between the beta and return of a security. Therefore, this method follows the Sharpe-Lintner model. Under this model, we assumed that the risk-free rate obtained is valid in order to simplify the analysis. Under this assumption, we are able to perform hypothesis testing on the market risk premium obtained. First, we tested if there is a relationship between the risk a security and its return. Secondly, we tested if the market risk premium obtained from the linear regression is sufficient to say that the market risk premium implied by the Philippine stock exchange index under the assumption is valid. Afterwards, we computed for a 95% confidence interval for the true market risk premium. Finally, we computed for the Pearson's R and the coefficient of determination to see whether the simple linear regression model is useful or not. However, this model can be restrictive since it assumes a uniform behavior in the entire market. To improve the model, we performed quadratic and cubic regression to be able to see how the market behaves at different levels of risk.

IV. Data Analysis

The aim of this paper was to determine the securities market line (SML) equation for the Philippine market. The SML equation describes the linear relationship between the beta β and required return μ_r of a security. The relationship is described by

$$\mu_r = r + MRP \times \beta,$$

where r is the risk free rate of the market and MRP is the market risk premium or the compensation for investors for taking one unit of risk. Because of such relationship between β and μ_r, a simple linear regression may be used to obtain the SML equation and see how the beta β of a security affects its required return μ_r.

In order to do so, data on the beta and returns of 30 securities were obtained from MarketWatch.com, an American financial information website, on November 28, 2019 [11]. The betas and returns of the 30 securities are shown in Table I below.

TABLE I. The Beta and Returns of Thirty Securities in the Philippine Market

Company	**Beta β**	**Return μ^***
JG Summit Holdings	1.37	36.98%
Banco De Oro	0.99	17.74%
SM Property Holdings, Inc.	1.24	9.36%
Alliance Global Group	0.89	-6.72%
Ayala Land, Inc.	1.21	10.10%
Jollibee Foods Corporation	0.75	-33.17%
Century Properties Group	0.6	37.21%
Bank of the Philippine Islands	0.76	-6.38%
Universal Robina Corporation	1.01	18.90%
Nickel Asia Corporation	0.98	48.64%
Metro Pacific Investments Corporation	0.92	-8.19%
LT Group, Inc.	0.58	-30.72%
ABS-CBN Corporation	0.25	-14.00%
Asia United Bank	0.98	34.75%
Vista Land & Lifescapesm Inc.	0.49	40.48%
Filinvest Land, Inc.	0.66	10.64%
Aboitiz Power Corporation	0.69	-1.85%
Manila Water Corporation	0.39	-34.51%
PLDT	1.02	-3.38%
Semirara Mining and Power Corp.	0.58	-5.21%
East West Bank	0.51	8.24%
Philippine National Bank	0.39	-7.35%
EEI Corporation	0.51	30.82%
Cosco Capital, Inc.	0.47	4.32%
2GO Group, Inc.	0.45	-28.59%
PhilWeb Corporation	0.24	-13.35%
NOW Corporation	1.09	-14.53%
Philex Petroleum Corporation	0.34	6.15%
PXP Energy Corporation	0.82	-29.74%
Benguet Corporation	0.12	-20.67%

After obtaining the required data, using a simple linear regression, the following were obtained.

$$S_{xx} = \sum_{i=1}^{30} \left(\beta_i - \bar{\beta}\right)^2 = 3.0386,$$

$$\widehat{MRP} = \frac{\sum_{i=1}^{30} \left(\beta_i - \bar{\beta}\right) \times \mu_{r_i}}{\sum_{i=1}^{30} \left(\beta_i - \bar{\beta}\right)^2} = 25.98\%,$$

$$\hat{r} = \overline{\mu_r} - \widehat{MRP} \times \bar{\beta} = -16.58\%.$$

Therefore, the implied SML equation from the linear regression is

$$\mu_r = -16.58\% + 25.98\% \times \beta.$$

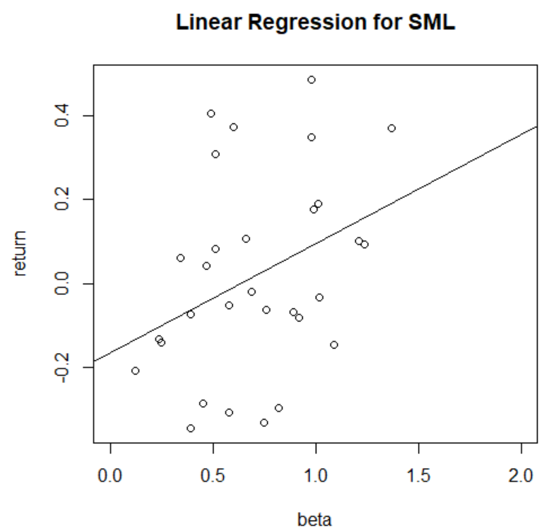

Fig. 1. Linear Regression for SML

Notice that the risk-free rate obtained from the model is negative. A possible explanation for this is because the Philippine market is performing poorly when the data was obtained. Global economic slowdown and rising tensions were dragging down the Philippine stock market [12]. However, for the purposes of this paper, we assumed that the risk-free rate implied by the model is valid and true. Because of this assumption, we focused the market risk premium implied by the model. From Table 1, we can calculate the implied MRP for each security using the formula

$$MRP^* = \frac{\mu^* - r}{\beta},$$

where μ^* is the observed return in the market. The results of this is shown in Table II below.

TABLE II. THE IMPLIED MARKET RISK PREMIUM OF THE THIRTY SECURITIES IN THE PHILIPPINE MARKET

Company	MRP^*
JG Summit Holdings	39.09%
Banco De Oro	34.67%
SM Property Holdings, Inc.	20.92%
Alliance Global Group	11.08%
Ayala Land, Inc.	22.05%
Jollibee Foods Corporation	-22.12%
Century Properties Group	89.65%
Bank of the Philippine Islands	13.42%
Universal Robina Corporation	35.13%
Nickel Asia Corporation	66.55%
Metro Pacific Investments Corporation	9.12%
LT Group, Inc.	-24.38%
ABS-CBN Corporation	10.32%
Asia United Bank	52.38%
Vista Land & Lifescapesm Inc.	116.45%
Filinvest Land, Inc.	41.24%
Aboitiz Power Corporation	21.35%
Manila Water Corporation	-45.97%
PLDT	12.94%
Semirara Mining and Power Corp.	19.60%
East West Bank	48.67%
Philippine National Bank	23.68%
EEI Corporation	92.94%
Cosco Capital, Inc.	44.47%
2GO Group, Inc.	-26.69%
PhilWeb Corporation	13.46%
NOW Corporation	1.88%
Philex Petroleum Corporation	66.85%
PXP Energy Corporation	-16.05%
Benguet Corporation	-34.08%

Using R, we can get the density plot of the MRP's and using the Shapiro-Wilks test for normality, we obtained the following results. The p-value of the Shapiro-Wilks test is 0.5778 which means that the MRP's are normally distributed. The histogram of the MRP's in Figure III also show that it is normally distributed.

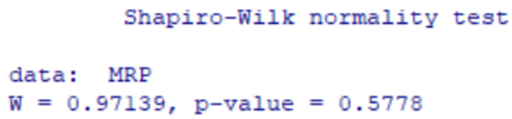

Fig. 2. Shapiro-Wilk Normality Test for MRP's Using R

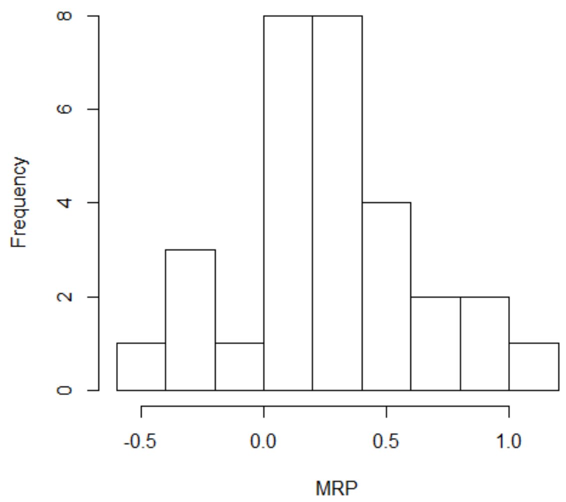

Fig. 3. Histogram of the MRP's

Furthermore, in the linear regression model, $\widehat{MRP} \sim N\left(MRP_0, \frac{\sigma^2}{S_{xx}}\right)$, where σ^2 is the market risk volatility and $S_{xx} = \sum_{i=1}^{30}(x_i - \bar{x})^2$. Therefore, we can test whether the beta β and required return μ_r has an association.

Let $H_0: MRP_0 = 0$ and $H_1: MRP_0 \neq 0$, and $\alpha = 0.10$. The test statistic is

$$t = \frac{\widehat{MRP}}{S_n/\sqrt{S_{xx}}} \sim t_{n-2}.$$

The rejection rule is $|t^*| \geq |t_{0.05,28}| = 1.701131$.

$$S_{yy} = \sum_{i=1}^{30}\left(\mu_{r_i} - \overline{\mu_r}\right)^2 = 1.6107,$$

$$S_n = \sqrt{\frac{1}{28} \times \sum_{i=1}^{30} \left(\mu_{r_i} - \widehat{\mu_r}\right)^2} = 0.2241,$$

$$t^* = \frac{25.98\%}{0.2241/\sqrt{3.0386}} = 2.0213.$$

Therefore, since $|t^*| \geq 1.701131$, we reject H_0 and $MRP_0 \neq 0$.

In addition, we can also test the MRP obtained in the linear regression against the implied MRP of the Philippine Stock Exchange index which has a beta of 1 and a return of 3.50% as of November 28, 2019 [13]. Therefore, assuming a risk-free rate of -16.58%, the implied MRP is $3.50\% + 16.58\% = 20.08\%$.

Let $H_0\colon MRP_0 = 20.08\%$ and $H_1\colon MRP_0 \neq 20.08\%$, and $\alpha = 0.10$. The test statistic is

$$t = \frac{\widehat{MRP} - MRP_0}{S_n/\sqrt{S_{xx}}} \sim t_{n-2}.$$

The rejection rule is $|t^*| \geq |t_{0.05,28}| = 1.701131$.

$$t^* = \frac{25.98\% - 20.08\%}{0.2241/\sqrt{3.0386}} = 0.4590538.$$

Therefore, since $|t^*| < 1.701131$, we do not reject H_0 and $MRP_0 = 20.08\%$.

Furthermore, we can get a 90% confidence interval for the MRP. The confidence interval is

$$\left(25.98\% - |t_{0.05,28}| \times \frac{0.2241}{\sqrt{3.0386}}, 25.98\% + |t_{0.05,28}| \times \frac{0.2241}{\sqrt{3.0386}}\right)$$
$$= (4.1103\%, 47.8497\%).$$

Finally, we can also get the Person's R of the linear regression model.

$$\hat{\rho} = \widehat{MRP} \times \frac{\sqrt{S_{xx}}}{\sqrt{S_{yy}}} = 25.98\% \times \sqrt{\frac{3.0386}{1.6107}} = 0.3568.$$

Moreover, the coefficient of determination is

$$R^2 = 0.3568^2 = 0.127,$$

which implied that the securities market line obtained from the linear regression model is not a useful model.

In order to improve this and prevent underfitting, we applied a quadratic regression of the data. Let $\mu_{r_i} = a\beta_i^2 + b\beta_i + c$. Using ordinary least squares, we have the following:

$$L(a,b,c) = \sum_{i=1}^{n} \left(\mu_{r_i} - \left(a\beta_i^2 + b\beta_i + c\right)\right)^2,$$

$$\frac{\partial L}{\partial a} = \sum_{i=1}^{n} -2\beta_i^2 \left(\mu_{r_i} - \left(a\beta_i^2 + b\beta_i + c\right)\right) = 0,$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^{n} -2\beta_i \left(\mu_{r_i} - \left(a\beta_i^2 + b\beta_i + c\right)\right) = 0,$$

$$\frac{\partial L}{\partial c} = \sum_{i=1}^{n} -2 \left(\mu_{r_i} - \left(a\beta_i^2 + b\beta_i + c\right)\right) = 0.$$

For brevity, let $\bar{\beta}^j = \sum \beta_i^j$. Thus,

$$\sum_{i=1}^{n} \mu_{r_i} \beta_i^2 = a\bar{\beta}^4 + b\bar{\beta}^3 + c\bar{\beta}^2,$$

$$\sum_{i=1}^{n} \mu_{r_i} \beta_i = a\bar{\beta}^3 + b\bar{\beta}^2 + c\bar{\beta}^1,$$

$$\sum_{i=1}^{n} \mu_{r_i} = a\bar{\beta}^2 + b\bar{\beta}^1 + cn,$$

To summarize, we express the system of equations in matrix form. We get

$$\begin{bmatrix} \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 \\ \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 \\ \bar{\beta}^2 & \bar{\beta}^1 & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \mu_{r_i} \beta_i^2 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i \\ \sum_{i=1}^{n} \mu_{r_i} \end{bmatrix}.$$

Therefore,

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 \\ \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 \\ \bar{\beta}^2 & \bar{\beta}^1 & n \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} \mu_{r_i} \beta_i^2 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i \\ \sum_{i=1}^{n} \mu_{r_i} \end{bmatrix},$$

provided that the 3×3 matrix is non-singular.

Using R, we get $a = 6.505953\%$, $b = 16.390043\%$, $c = -13.7097.15\%$ and the quadratic regression is

$$\mu_{r_i} = 6.50\%\beta_i^2 + 16.39\%\beta_i - 13.71\%.$$

```
       [,1]
[1,]  0.06505953
[2,]  0.16390043
[3,] -0.13709715
```

Fig. 4. The Values of a, b, c computed Using R

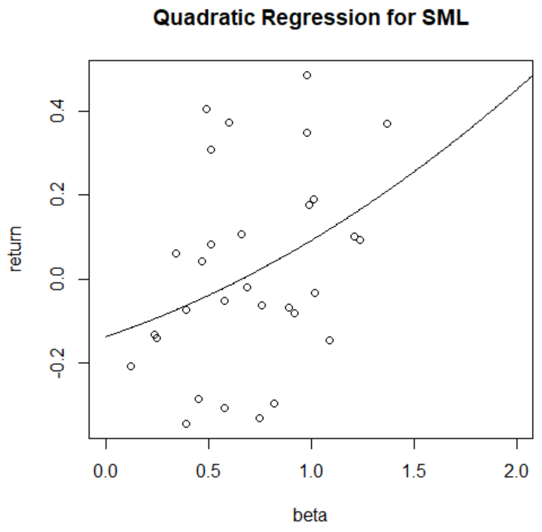

Fig. 5. Quadratic Regression for SML

The results show that the quadratic regression is convex which means that the market has an increasing marginal return.

In Figure 5, it seems that the quadratic regression is underfitted. We further improved this analysis by considering a cubic regression. Following the analysis of quadratic regression, we can extend the formula for finding the coefficient as follows. If $\mu_{r_i} = a\beta_i^3 + b\beta_i^2 + c\beta_i + d$, then,

$$\begin{bmatrix} \bar{\beta}^6 & \bar{\beta}^5 & \bar{\beta}^4 & \bar{\beta}^3 \\ \bar{\beta}^5 & \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 \\ \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 \\ \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 & n \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \mu_{r_i} \beta_i^3 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i^2 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i \\ \sum_{i=1}^{n} \mu_{r_i} \end{bmatrix}$$

or equivalently,

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \bar{\beta}^6 & \bar{\beta}^5 & \bar{\beta}^4 & \bar{\beta}^3 \\ \bar{\beta}^5 & \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 \\ \bar{\beta}^4 & \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 \\ \bar{\beta}^3 & \bar{\beta}^2 & \bar{\beta}^1 & n \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} \mu_{r_i} \beta_i^3 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i^2 \\ \sum_{i=1}^{n} \mu_{r_i} \beta_i \\ \sum_{i=1}^{n} \mu_{r_i} \end{bmatrix}$$

provided that the 4×4 matrix is non-singular.

Using R, we get $a = 103.21245\%$, $b = -223.19772\%$, $c = 164.66822\%$, $d = -39.70105\%$ and the cubic regression is

$$\mu_{r_i} = 103.21\%\beta_i^3 - 223.20\beta_i^2 + 164.67\%\beta_i - 39.70\%.$$

```
       [,1]
[1,]  1.0321245
[2,] -2.2319772
[3,]  1.6466822
[4,] -0.3970105
```

Fig. 6. The Values of a, b, c, d computed Using R

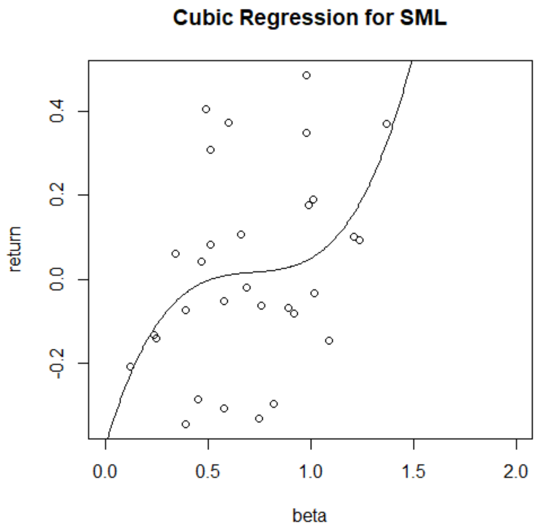

Fig. 7. Quadratic Regression for SML

In Figure 7, we see that a point near $\beta = 1$ seems to be a point of inflection on the securities market line. If $0 < \beta < 1$, the security is said to be less risky. Securities having low risk has a concave SML. Therefore, it exhibits a decreasing maginal return. On the other hand, if $\beta > 1$, the security is said to be risky. Securities having high risk has a convex SML. Therefore, it exhibits an increasing maginal return.

Overall, we can see that as we increase the degree of the polynomial regression, the equation becomes more fitted and it gives us a better picture of the movement of the securities as shown in Figure 8. The linear regression is underfitted since it assumes constant returns. Moreover, the quadratic regression is unfitted since it assumes uniform movement of securities in the market. However, the cubic regression gives us a sufficient information since it starts to segregate the securities into groups which exhibits different returns. Theoretically, this could be extended to an even higher degree polynomial but for practical purposes, a cubic regression already suffices.

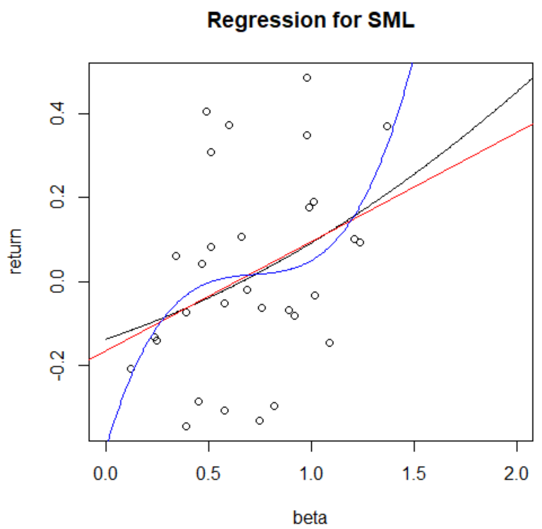

Fig. 8. Linear, Quadratic, Cubic Regression for SML

V. Summary and Conclusion

This paper aimed to find the securities market line (SML) equation for the Philippine market by gathering the returns and betas of 30 securities in the market. Afterwards, using a linear regression model, the estimated SML equation was obtained to be

$$\mu_r = -16.58\% + 25.98\% \times \beta.$$

In addition, we assumed that the risk-free rate obtained here is valid. Given this assumption, we tested the implied $MRPs$ of each stock for normality using the Shapiro-Wilks test which obtained a result indicating that β is normal. As such, we were able to perform hypothesis testing on the market risk premium obtained by the model. First, the MRP was tested with $H_0\!: MRP = 0$ and $H_1\!: MRP \neq 0$ with $\alpha = 0.10$. Since we do not know the true volatility of the market, we perform t-test. This test lead to the decision of rejecting H_0 or equivalently that the MRP is not equal to 0. Moreover, we also tested the MRP with $H_0\!: MRP = 20.08\%$, obtained from the Philippine Stock Exchange index, and $H_1\!: MRP \neq 20.08\%$ with $\alpha = 0.10$. This test lead to the decision of not rejecting H_0 or equivalently, that the MRP is 20.08%. Afterwards, we obtained a 95% confidence interval of

$$(4.1103\%, 47.8497\%).$$

Finally, we computed the coefficient of determination of the model and obtained a value of 0.127 which implies that the model is not useful.

In order to improve the model and prevent underfitting (and overgeneralizing the market), we performed a quadratic regression on the data obtained. This model has an estimated SML equation of

$$\mu_{r_i} = 6.50\%\beta_i^2 + 16.39\%\beta_i - 13.71\%.$$

The equation tells us that from the data obtained, there seems to be an increasing marginal return. However, despite the improvement, there are still limitations since it generalizes the market to only on trend.

In order to further improve the model, we performed a cubic regression on the data obtained. This model has an estimated SML equation of

$$\mu_{r_i} = 103.21\%\beta_i^3 - 223.20\beta_i^2 + 164.67\%\beta_i - 39.70\%.$$

The equation tells us that from the data obtained, there seems to be a decreasing marginal return when the security is less risky and an increasing marginal return when the security is risky. The analysis could still be extended to higher polynomial regression, but it is not practical anymore.

Overall, the results of this paper show that the original SML equation is not reflective and representative of the market since its coefficient of determination is small. Therefore, the SML equation is not a useful model and its purpose only remains in giving investors a benchmark of what the appropriate return is; however, they should take this with a grain of salt since the model is flawed. The model is limited since it does not consider other variables, such as the state of the economy, inflation, government policies and interventions among others, which could have a significant impact on the financial market. With these, we recommend future studies (1) consider the aggregate market rather than considering only a small sample to have a better view on the movements of the securities, (2) to analyze the movements of each securities at different times, and (3) to widen the variables considered to have a better model for the market.

VI. References

[1] Treynor, Jack L., Market Value, Time, and Risk (August 8, 1961). Available at http://dx.doi.org/10.2139/ssrn.2600356

[2] The Library of Economics and Liberty. (n.d.). William F. Sharpe. Retrieved from https://www.econlib.org/library/Enc/bios/Sharpe.html.

[3] Fama, E. F., & French, K. R. (2003). The Capital Asset Pricing Model: Theory and Evidence. *SSRN Electronic Journal*, *18*(3), 25–46. doi: 10.2139/ssrn.440920

[4] Ernest, C., & St, L. S. (2014). Cost of equity in the Black Capital Asset Pricing Model.

[5] Roll, Richard. (1978). The Journal of Finance. Retreived from https://sci-hub.tw/https://onlinelibrary.wiley.com/doi/abs /10.1111/j.1540-6261.1978.tb02047.x

[6] Jensen, M. C., & Scholes, M. (1972). The Capital Asset Pricing Model: Some Empirical Tests. Retreived from http://www.efalken.com/LowVolClassics/blackjensenscholes.pdf?fbclid=IwAR0-bSCuGR1JJRYiOnPUySc7XkS9X18xUieqLUDn8m1EPpaMlLT9KsHnZXM

[7] Dzaja, J. ,& Aljinovic, Z. (2010). Testing CAPM Model on the Emerging Markets of the Central and Southeastern Europe. Retrived from https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=13&ved=2ahUKEwi885fZ75nmAhVZfXAKHY_CA6UQFjAMegQIAhAC&url=https%3A%2F%2Fhrcak.srce.hr%2Ffile%2F143353&usg=AOvVaw3a7H2bVefKiaOAjtey_VrB

[8] Chen, Z. ,& Zhang, P. (2007). The CAPM Applicability Study on Chinese Stock Markets. Retrieved from https://sci-hub.tw/https://ieeexplore.ieee.org/document /5073172

[9] Garg, V. (1998). The Capital Asset Pricing Model: A Test on the Stock Exchange of Singapore. Retrieved from https://researchdirect.westernsydney.edu.au/islandora/object/uws%3A459/datastream/PDF/view

[10] Curran, M. ,& Velic, A. (2018). The CAPM, National Stock Market Betas, and Macroeconomic Covariaes: A Global Analysis. Retrieved from https://www.tcd.ie/Economics/TEP/2018/tep0618.pdf

[11] MarketWatch. (2016). *MarketWatch: Stock Market News - Financial News*. Available at: http://www.marketwatch.com.

[12] Gonzales, I. (2019, October 1). Global concerns continue to weigh down local stocks. Retrieved from https://www.philstar.com/business/2019/10/02/1956576/global-concerns-continue-weigh-down-local-stocks.

[13] PSEi Index. (n.d.). Retrieved from https://www.barrons.com/quote/index/ph/philippine stock exchange/psei?fbclid=IwAR3xF1NfgUSJ396OAJg1fU6h9-MJHNOHiev2ylDXD3grQghaRkuzC9ZRdfM.

VII. Appendix

```
Call:
lm(formula = return ~ beta)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.36075 -0.13290 -0.03447  0.11196  0.44330 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept)  -0.1658     0.1000  -1.658   0.1085  
beta          0.2598     0.1285   2.021   0.0529 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2241 on 28 degrees of freedom
Multiple R-squared:  0.1273,	Adjusted R-squared:  0.09617 
F-statistic: 4.086 on 1 and 28 DF,  p-value: 0.0529
```

A: R Summary of the Linear Regression

```
INPUTS
beta=c(1.37,0.99,1.24,0.89,1.21,0.75,0.6,0.76,1.01,0.98,0.92,0.58,0.25,0.98,0.49,0.66,0
return=c(0.3698,0.1774,0.0936,-0.0672,0.101,-0.3317,0.3721,-0.0638,0.189,0.4864,-0.0819

TESTING FOR NORMALITY
MRP=c(0.390948905,0.346666667,0.209193548,0.110786517,0.220495868,-0.2212,0.8965,0.1342
shapiro.test(MRP)
hist(MRP)

LINEAR REGRESSION
sxx=sum((beta-mean(beta))^2)
syy=sum((return-mean(return))^2)
sxy=sum((beta-mean(beta))*(return-mean(return)))
mrp=sxy/sxx
rrf=mean(return)-mrp*mean(beta)
e=(return-(rrf+mrp*beta))
Sn2=sum(e^2)/(length(beta)-2)
summary(lm(return~beta))

TEST FOR LINEAR REGRESSION
t0=(mrp)/sqrt(Sn2/sxx)
t1=(mrp-0.2008)/sqrt(Sn2/sxx)
qt(0.05,28)

PLOTTING LINEAR REGRESSION
plot(beta,return,lim=c(-1,10), xlim=c(0,2),main="Linear Regression for SML",type="p")
abline(lm(return~beta))
```

B: R Script for the Inputs and Linear Regression

```
QUADRATIC REGRESSION
b4=sum(beta^4)
b3=sum(beta^3)
b2=sum(beta^2)
b1=sum(beta)
ub2=sum(return*beta^2)
ub1=sum(return*beta)
u=sum(return)

A=matrix(c(b4,b3,b2,b3,b2,b1,b2,b1,30),nrow=3,ncol=3,TRUE)
B=matrix(c(ub2,ub1,u),3,1,TRUE)
C=solve(A)%*%B

PLOTTING QUADRATIC REGRESSION
q=function(x) 0.06505953*x^2+0.16390043*x-0.13709715
a=seq(0,20,0.005)
plot(beta,return,lim=c(-1,10), xlim=c(0,2),main="Quadratic Regression for SML",type="p")
lines(a,q(a))
```

C: R Script for the Quadratic Regression

```
CUBIC REGRESSION
b6=sum(beta^6)
b5=sum(beta^5)
ub3=sum(return*beta^3)

A'=matrix(c(b6,b5,b4,b3,b5,b4,b3,b2,b4,b3,b2,b1,b3,b2,b1,30),nrow=4,ncol=4,TRU
B'=matrix(c(ub3,ub2,ub1,u),4,1,TRUE)
C'=solve(A)%*%B

PLOTTING CUBIC REGRESSION
p=function(x)  1.0321245*x^3-2.2319772*x^2+ 1.6466822*x-0.3970105
a=seq(0,20,0.005)
plot(beta,return,lim=c(-1,10), xlim=c(0,2),main="Cubic Regression for SML",typ
lines(a,p(a))
```

D: R Script for the Cubic Regression